ACOUSTIC APPROXIMATION IN THE PENETRATION OF AN IDEAL FLUID BY A BLUNTED CONTOUR

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The description of the unsteady motion of a fluid occupying a region which changes in time and whose boundary consists of a free surface and a rigid wall, and the line of contact between them is of considerable interest, from both the theoretical and applied hydrodynamics points of view. The law of motion of the rigid wall is given, and all boundaries of the fluid are free at the initial moment. The topology of the flow changes at the initial moment, and a rigid part of the boundary appears, which was previously absent. Specification of the problem makes it necessary to simultaneously determine both the motion of the fluid and the position of the line of contact at each moment in time. Even after significant simplifications, the problem remains complex, and exact results are virtually nonexistant.

The initial stages of the process are of special interest. A detailed review of works devoted to the study of this problem are given in [1, 2]. A new approach to the analysis of the initial stages of the motion of the fluid was proposed in [3]. This approach is based on the introduction of Lagrangian coordinates in which the region of the flow is fixed. In the present work, this approach is used to analyze the plane problem of the symmetric penetration by a rigid parabolic contour of an ideal, slightly compressible fluid.

1. Problem Statement. We examine the plane, unsteady motion of an ideal, slightly compressible fluid, which occupies the half-plane $y' < 0$ at time $t' = 0$, and is at rest (here the prime denotes dimensional variables). The line $y' = 0$ at the initial moment is a free boundary.

Let R, V be positive constants. For fixed t', the equation

$$y' = x'^2/(2R) - Vt' \tag{1.1}$$

determines the parabola in the x', y' plane, which is identified with a rigid, nondeformable contour. At $t' = 0$, it touches the free boundary at the point $x' = 0$. Relation (1.1) gives the motion of the contour along the y' axis with constant velocity V. We must find the motion of the fluid which arises from this, assuming that the part of its boundary which is not part of the rigid contour remains free. This assumption, in particular, means that in the plane of the Lagrangian coordinates ξ', η', the region occupied by the fluid is known beforehand: it is the half-plane $\eta' < 0$. In the Lagrangian variables ξ', η', the unknown functions are the pressure p', the density ρ' and also the coordinates x', y' of the fluid parcel which occupied the position ξ', η' at $t' = 0$. The vector displacement of the fluid parcel $\mathbf{X}'$ is determined from the equation $\mathbf{X}' = \mathbf{x}' - \boldsymbol{\xi}'$, where $\mathbf{x}' = (x', y')$, $\boldsymbol{\xi}' = (\xi', \eta')$, $\mathbf{X}' = (X', Y')$.

Before fomulating the initial boundary value problem in dimensionless variables, we examine the issue of the choice of length and time scales and the unknown functions. From (1.1) we find that the law of motion of a point intersecting the contour from the undisturbed surface of the fluid ($y' = 0$) is given by $x_1'(t') = \sqrt{2RVt'}$. Consequently, directly after the start of penetration by the contour, the point moves with a velocity which is known to exceed the local speed of sound, and the free surface remains undisturbed. This stage is called the supersonic stage, and for low penetration velocities, its duration T_s can be estimated by equating the velocity of motion of the point x_1' with the speed of sound in the fluid at rest c_0, from which we have $T_s \approx RV/(2c_0^2)$. Correspondingly, it is convenient to select the quantity $M^2(R/V)$ ($M = V/c_0$ is the Mach number) for the time scale, and the quantity RM as the length scale, which is of the same order of magnitude as the distance traversed by the point x_1' in the course of the supersonic stage. As a consequence of problem symmetry, the displaement vector of the fluid parcel with coordinates $\xi' = 0$, $\eta' = 0$ is known beforehand [$\mathbf{X}' = (0, -Vt')$], and therefore M^2R is chosen for the displacement scale. The density of the fluid at rest ρ_0 is chosen as the density scale. From the law of conservation of momentum it follows

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that it is necessary to take the one-dimensional shock pressure $\rho_0 c_0 V$ as the pressure scale. We transform to dimensionless variables, whose designation is distinguished by the absence of a prime.

Written in Lagrangian coordinates, the Euler equations have the form

$$\rho J^* \mathbf{X}_{tt} + \nabla_\xi p = 0, \quad \rho |J| = 1 \quad (\eta < 0). \tag{1.2}$$

The first of these is the momentum equation, the second is the continuity equation. It is necessary to add an equation of state to (1.2), which for a slightly compressible fluid with relatively small pressures can be represented as [2]

$$p = (\rho^n - 1)/(n\mathrm{M}) \tag{1.3}$$

(for water, $n \approx 7.15$). In (1.2), $J = \partial(\mathbf{x})/\partial(\xi)$ is the Jacobian; J^* is the adjoint to J.

We must find the solution to (1.2), (1.3) in the region $\eta < 0$, $t > 0$ for initial conditions

$$\mathbf{X} = 0, \mathbf{X}_t = 0 \quad (t = 0, \eta \leqslant 0), \tag{1.4}$$

which are satisfied everywhere except possibly at the origin, and the boundary conditions

$$p = 0 \ (\eta = 0, \quad |\xi| > a(t)); \tag{1.5}$$

$$Y = (1/2)(\xi + \mathrm{M}X)^2 - t \quad (\eta = 0, \ |\xi| < a(t)) \tag{1.6}$$

[a(t) is the inverse image in Lagrangian coordinates of the point of contact of the free boundary of the fluid with the surface of the rigid body]. Condition (1.6) signifies that a fluid parcel incident on some part of the rigid boundary can only be displaced along that boundary.

Additionally we require that the kinetic energy of the fluid T, defined by the expression

$$T = (\rho_0 V^2/2) \int_{-\infty}^{0} \int_{-\infty}^{\infty} |\mathbf{X}_t|^2 d\xi \, d\eta,$$

be finite at all times of the motion

$$T < +\infty \quad (t > 0). \tag{1.7}$$

In addition, we are interested only in those solutions to problem (1.2)-(1.7) which satisfy the one-sided inequality

$$\mathrm{M}^{-1}\eta + Y \leqslant (1/2)(\xi + \mathrm{M}X)^2 - t \quad (\eta \leqslant 0, -\infty < \xi < +\infty, t > 0). \tag{1.8}$$

This signifies that the fluid parcels cannot be found in the "forbidden" region bounded by contour (1.1). The supplementary conditions (1.7), (1.8), applicable to the problem of penetration in Pukhnachev [3].

The approximate solution to problem (1.2)-(1.8) will be sought for small M. If in (1.2)-(1.8) we formally go to the limit $M \to 0$ while accounting for the equality $J = I + \mathrm{M}\partial(\mathbf{X})/\partial(\xi)$ (I is the unit matrix), then the solution of the limiting problem gives the leading term of the asymptote of the unknown functions for $M \ll 1$. Keeping the previous notation for the limits of the unknown functions as $M \to 0$, we find from (1.2) and (1.3)

$$\mathbf{X}_{tt} + \nabla_\xi p = 0; \tag{1.9}$$

$$p = -\mathrm{div}_\xi \, \mathbf{X}. \tag{1.10}$$

The momentum equation (1.9) and initial conditions (1.4) show that there exists a displacement potential $\Phi(\xi, \eta, t)$ such that $\mathbf{X} = \nabla_\xi \Phi$. Substituting (1.10) into (1.9) and considering that the potential Φ is defined to within an arbitrary, time-dependent term, we find that the function Φ must satisfy the wave equation

$$\Phi_{tt} = \Phi_{\xi\xi} + \Phi_{\eta\eta} \quad (\eta < 0). \tag{1.11}$$

The inpenetrability condition (1.6) for M = 0 has the form

$$\Phi_\eta = (1/2)\xi^2 - t \quad (\eta = 0, \ |\xi| < a(t)). \tag{1.12}$$

Using (1.11), Eq. (1.10) gives $p = -\Phi_{tt}$. After a double integration over t, (1.5) at the free boundary is represented as

$$\Phi = 0 \quad (\eta = 0, \ |\xi| > a(t)). \tag{1.13}$$

Equation (1.7) gives

$$\int_{-\infty}^{0}\int_{-\infty}^{\infty} |\nabla_\xi \Phi_t|^2 d\xi\, d\eta < +\infty \quad (t > 0), \tag{1.14}$$

the one-sided inequality (1.8) is written in the form

$$\Phi_\eta \leqslant (1/2)\xi^2 - t \quad (\eta = 0, -\infty < \xi < +\infty). \tag{1.15}$$

It is necessary to add to (1.11)-(1.15) the initial conditions

$$\Phi = 0, \Phi_t = 0 \quad (\eta \leqslant 0, t = 0). \tag{1.16}$$

System (1.11)-(1.16) is the acoustic approximation to the problem of the penetration of a parabolic contour into an ideal, slightly compressible fluid. In problems of penetration by a rigid body, the acoustic approximation is usually written in Eulerian variables [4]. In order to transform from the proposed statement of the problem to the usual statement, it is necessary to formally differentiate relations (1.11)-(1.13) by t, and determine the potential velocity $\varphi = \Phi_t$. In addition, one must bear in mind that the Eulerian x and Lagrangian variables ξ differ from one another by a quantity of the order $O(M^2)$ in the considered time interval as $M \to 0$, which makes it possible to equate these two variables.

Note that although the relations in (1.11)-(1.16) are linearized, the problem remains nonlinear, since it is necessary to find not only the potential of the displacement, but also the function a(t), which gives the position of the points of contact. This circumstance makes the statement of the problem in the form (1.11)-(1.16) preferable. Recall that in Eulerian variables, the position of the points of contact is determined from the so-called Wagner condition [5]

$$\int_0^t \varphi_y(a(t), 0, \tau)\, d\tau = \frac{1}{2} a^2(t) - t \quad (t > 0),$$

which is a nonlinear integral equation in terms of the function a(t). The author was unable to simplify to any degree this equation. In the proposed problem statement, a(t) is the root of an algebraic equation in simple cases, and the root of a transcendental equation in complex cases. For certain examples, the function a(t) is determined in explicit form. On the other hand, if the function a(t) is known, then the pressure distribution along the wetted part of the contour and the velocity field are conveniently computed from a statement of the problem in Eulerian variables.

2. Auxiliary Problem. We examine the initial boundary problem with mixed boundary conditions

$$\begin{gathered} \psi_{tt} = \psi_{\xi\xi} + \psi_{\eta\eta} \quad (\eta < 0), \psi = 0 \quad (\eta = 0, |\xi| > a(t)), \\ \psi_\eta = f(\xi, t) \quad (\eta = 0, |\xi| < a(t)), \psi = \psi_t = 0 \quad (\eta \leqslant 0, |\xi| > 0, t = 0), \\ |\nabla_\xi \psi| \to 0 \quad (\xi^2 + \eta^2 \to \infty) \end{gathered} \tag{2.1}$$

with the following restrictions: a) $f(-\xi, t) = f(\xi, t)$, $a(0) = 0$; b) $a'(t_*) = 1$, $a'(t) > 1$ for $0 < t < t_*$, $0 < a'(t) < 1$ for $t > t_*$; c) a(t) is a piecewise-smooth function, $f(\xi, t)$ is a smooth function of its arguments. We must determine $\psi(\xi, 0, t)$ for $|\xi| < a(t)$ and $\psi_\eta(\xi, 0, t)$ for $|\xi| > a(t)$, assuming that the functions $f(\xi, t)$ and a(t) are given.

The solution to such a problem is based on the use of an integral relation which couples the boundary values $\psi(\xi, 0, t)$ and $\psi_\eta(\xi, 0, t)$ [6]:

$$\psi(\xi, 0, t) = \frac{1}{\pi} \iint_{\sigma(\xi, t)} \frac{\psi_\eta(\xi_1, 0, t_1)\, d\xi_1 dt_1}{\sqrt{(t - t_1)^2 - (\xi - \xi_1)^2}}. \tag{2.2}$$

Here $\sigma(\xi, t)$ is the isosceles right triangle in the half-plane ξ_1, t_1 ($t_1 > 0$) with vertex at the point (ξ, t) and its base on the $O\xi_1$ axis (Fig. 1). For $0 < t < t_*$, the expansion of the disturbed region [$\eta = 0$, $|\xi| < a(t)$] takes place with a velocity which exceeds the propagation velocity of the disturbance [for problem (2.1), this velocity is equal to unity] in the medium. Therefore, the right curve OAC retains the initial conditions, i.e., $\psi_\eta(\xi, 0, t) \equiv 0$ on this curve. In the region $|\xi| < a(t)$, the function $\psi_\eta(\xi, 0, t)$ is known, and therefore, if it is found in the region BAC, then the problem is solved.

We write (2.2) for the point P, which lies strictly inside the region BAC, but is below the line $t_1 = \xi_1 + t_* + a(t_*)$. Then the integration in (2.2) is carried out over two regions:

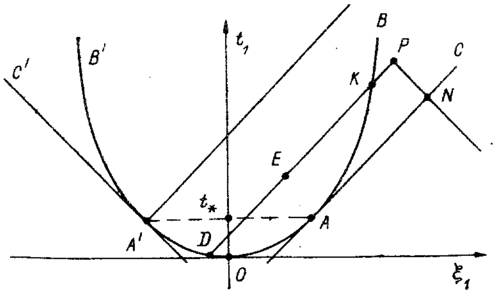

Fig. 1

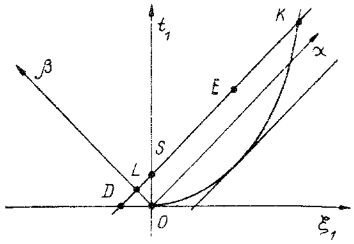

Fig. 2

σ_1 = KAOD, where $\psi_\eta(\xi_1, 0, t_1) = f(\xi_1, t_1)$, and σ_2 = PKAN, in which $\psi_\eta(\xi, 0, t_1)$ must be determined. From the condition $\psi(\xi, 0, t) = 0$ for $|\xi| > a(t)$ and (2.2), we obtain the integral equation

$$\iint_{\sigma_2} \frac{\psi_\eta(\xi_1, 0, t_1)\, d\xi_1\, dt_1}{\sqrt{(t-t_1)^2-(\xi-\xi_1)^2}} = -\iint_{\sigma_1} \frac{f(\xi_1, t_1)\, d\xi_1\, dt_1}{\sqrt{(t-t_1)^2-(\xi-\xi_1)^2}},$$

the solution of which is constructed in [6] and has the form

$$\psi_\eta(\xi, 0, t) = -\frac{1}{\pi\sqrt{|PK|}} \int_0^{|DK|} \frac{\sqrt{\mu}\, f(E)}{|PK|+\mu}\, d\mu, \tag{2.3}$$

where |PK| and |DK| are the lengths of the segments PK and DK, respectively; f(E) is the value of the function $f(\xi, t)$ at the point E, which lies on the segment DK, such that $|EK| = \mu$ (see Fig. 1). Formula (2.3) is also valid in the case where the point P lies somewhat above the line $t_1 = \xi_1 + t_* + a(t_*)$. In this case, the point D lies on the line A'C' and (2.3) contains the integral along the segment which lies in the region B'A'C' below the line $t_1 = -\xi_1 + t_* + a(t_*)$, in which $f(E) = \psi_\eta(\xi_1, 0, t_1)$, where the right-hand side is a consequence of the symmetry of the problem computed using (2.3). Subsequent use of (2.3) makes it possible to construct $\psi_\eta(\xi, 0, t)$ over the entire region BAC, which in fact solves the auxiliary problem.

3. The Law of Motion of the Point of Contact. If in (2.1), we set $\psi = \Phi(\xi, \eta, t)$, $f(\xi, t) = \xi^2/2 - t$, then we arrive at the acoustic approximation in the penetration problem (1.11)-(1.16), written in Lagrangian variables. Correspondingly, for $\psi = \varphi(x, y, t)$, $\xi = x$, $\eta = y$, $f(x, y) = -1$ we have the same approximation, but in Eulerian coordinates.

In the Lagrangian description, the left-hand side of (2.3) is equal to the rise of the fluid parcel which lies on the free boundary, at the initial moment, to a distance ξ from the origin, above the undisturbed level of the fluid. But with the approach of the point of observation P to the point of contact K, i.e., for $|PK| \to 0$, the right-hand side of (2.3) tends toward infinity in the general case, and condition (1.15) cannot be satisfied. Consequently, it is necessary to require that for $|PK| = 0$, the integral in (2.3) be equal to zero:

$$\int_0^{|DK|} \mu^{-1/2} f(E)\, d\mu = 0. \tag{3.1}$$

This fundamental equation serves to compute the quantity |DK|, knowledge of which is sufficient to determine the function a(t). It turns out that when (3.1) is satisfied, displacement of the fluid parcels is described right up to the boundary of the fluid volume by continuous functions, for which conditions (1.14), (1.15) are satisfied.

We determine the function a(t) directly after the emergence of the disturbance wave front at the boundary ($t_* = 1/2$, $t > t_*$). We introduce a new coordinate system α, β which has its origin in common with the old system ξ_1, t_1, but is rotated counterclockwise with respect to the latter by an angle of $\pi/4$. In this case, $\xi_1 = (\alpha - \beta)/\sqrt{2}$, $t_1 = (\alpha + \beta)/\sqrt{2}$. In the new system, let the point D have the coordinates (α_0, β_0), which can be assumed to be known. Then the point E has the coordinates $(\alpha_0 + |DK| - \mu, \beta_0)$. After introducing the new integration variable $\sigma = |DK| - \mu$, we find

$$f(E) = (1/4)\sigma^2 + c\sigma,\ c = (\alpha_0 - \beta_0)/2 - 1/\sqrt{2}, \tag{3.2}$$

where we have used $f(D) = 0$. Substituting (3.2) into (3.1) and computing the integral, we obtain the formula $|DK| = -5c$. The formulas for transforming from one coordinate system to the other give

$$a(t) = (\alpha_0 - \beta_0 + |DK|)/\sqrt{2}, \quad t = (\alpha_0 + \beta_0 + |DK|)/\sqrt{2}. \tag{3.3}$$

This system determines the function a(t) in parametric form. We choose as the parameter the quantity $k = \beta_0 + 2^{-3/2}$. Note that k is equal to zero when the point D coincides with point A, and increases with removal of the point D from point A along the curve AOA'. The parameter k takes on its maximum value of $\sqrt{2}$ when D coincides with A'.

We denote the difference $\alpha_0 - \beta_0$ by ω and determine the dependence $\omega = \omega(k)$ from the equation $f(D) = 0$. Straightforward calculations give $\omega = \sqrt{2} - 2^{5/4}k^{1/2}$, from which we have $c = -2^{1/4}k^{1/2}$. System (3.3) takes on the form

$$a(t) = 1 + 3 \cdot 2^{-1/4}k^{1/2}, \quad t = a(t) - 1/2 + \sqrt{2}k, \tag{3.4}$$

from which it follows that $a^2(t) + (5/2)a(t) - 5/4 = (9/2)t$. Thus, in this case, a(t) is given in explicit form

$$a(t) = (3\sqrt{5 + 8t} - 5)/4. \tag{3.5}$$

We determine the time t_s at which the rarefaction wave, formed at the moment $t_* = 1/2$ from the left side of the contour, reaches the right contact point. To do this, it is necessary to set $k = \sqrt{2}$ in system (3.4); then $a(t_s) = 4$, and $t_s = 11/2$. This means that when $1/2 < t < 11/2$, the law of motion of the contact point is given by (3.5). For $t > 11/2$, the function a(t) has a more complex form, and at present it is not known if it is possible to write it in analytical form.

From (3.5), it is possible to analyze the change in the law of motion of the contact point at the moment the wave front of the disturbance emerges at the free boundary. It is straightforward to obtain the following relations: $a(1/2 + 0) = a(1/2 - 0) = 1$, $a'(1/2 + 0) = a'(1/2 - 0) = 1$, $a''(1/2 + 0) = -4/9$, $a''(1/2 - 0) = -1$. Thus, the function a(t) is continuously differentiable in the interval $0 < t < 11/2$, and its second derivative changes discontinuously at $t = 1/2$, with $a''(1/2 + 0) - a''(1/2 - 0) = 5/9$, i.e., at this moment in time, the point of contact is accelerated.

4. Model Problem. We examine the plane problem of penetration of a semi-infinite plate with rounded edges in the acoustic approximation. The initial boundary value problem for the displacement potential $\Phi(\xi, \eta, t)$ has the form

$$\begin{gathered}
\Phi_{tt} = \Phi_{\xi\xi} + \Phi_{\eta\eta} \quad (\eta < 0, -\infty < \xi < +\infty),\\
\Phi_\eta = (1/2)\xi^2 - t \quad (\eta = 0, 0 < \xi < a(t)),\\
\Phi_\eta = -t \quad (\eta = 0, \xi < 0),\\
\Phi = 0 \quad (\eta = 0, \xi > a(t)),\\
\Phi = \Phi_t = 0 \quad (\eta \leqslant 0, t = 0), \; |\nabla_\xi \Phi| \to 0 \; (\xi \to +\infty, \eta \leqslant 0),\\
\Phi_\eta \leqslant (1/2)\xi^2 - t \quad (\eta = 0, \xi > 0).
\end{gathered} \tag{4.1}$$

The condition of finite kinetic energy of the fluid (1.14), which we know cannot be satisfied in this model problem, is not included in problem statement (4.1).

In the formulated problem, there is a single point of contact, and therefore its law of motion is simpler than in a penetration problem with a parabolic contour (1.11)-(1.16), and can be given in explicit form. Just as before, the free surface of the fluid remains undisturbed until the moment $t = 1/2$, and consequently, $a(t) = \sqrt{2t}$ for $0 < t < 1/2$. Furthermore, up until some moment t_1, the law of motion of the point of contact will be the same as in the problem of symmetric penetration by the parabolic contour. For $1/2 < t < t_1$, the function a(t) is given by formula (3.5). The value t_1 is determined from the condition $t_1 = a(t_1)$ (Fig. 2) and is equal to 5/2.

Let us find the law of motion of the point of contact for $t > 5/2$. To do this, we will start from condition (3.1) and use Fig. 2. As in Paragraph 3, we introduce a new integration variable $\sigma = |DK| - \mu$. It is clear that $\sigma = |DE|$. When the point E lies inside the interval DS, then $f(E) = -t$, but $t = \sigma/\sqrt{2}$. From this we have $f(E) = -\sigma/\sqrt{2}$ for $0 < \sigma < 2\beta_0$, $\beta_0 = |OL|$, where β_0 is taken as a parameter. When the point E lies inside the interval SK, then $f(E) = \xi^2/2 - t$. Here $t = \sigma/\sqrt{2}$, $\xi = \sigma/\sqrt{2} - |DO|$, $|DO| = \sqrt{2}\beta_0$, from which we have $f(E) = (\sigma/\sqrt{2} - \sqrt{2}\beta_0)^2/2 - \sigma/\sqrt{2}$ for $2\beta_0 < \sigma < |DK|$. Condition (3.1) gives

$$-\frac{1}{\sqrt{2}}\int_0^{2\beta_0}\frac{\sigma d\sigma}{\sqrt{|DK| - \sigma}} + \int_{2\beta_0}^{|DK|}\frac{\sigma^2/4 - \sigma(\beta_0 + 2^{-1/2}) + \beta_0^2}{\sqrt{|DK| - \sigma}}d\sigma = 0. \tag{4.2}$$

Equation (4.2) serves to determine $|DK|$ as a function of β_0 ($\beta_0 > 0$). If $|DK|$ is known, then the coordinates of the point K are:

$$\xi_K = |DK|/\sqrt{2} - \sqrt{2}\beta_0,\ t_K = |DK|/\sqrt{2}. \tag{4.3}$$

System (4.3) gives the function a(t) [$\xi_K = a(t_K)$] in parametric form for $t > 5/2$.

We make the substitution $\sigma = |DK|\nu$ in (4.2) and denote the quantity $\beta_0/|DK|$ by ω, and $\sqrt{1 - 2\omega}$ by b. After computing the integrals in (4.2), we find $|DK| = 5/(\sqrt{2}b^5)$, $\beta_0 = 5/(2\sqrt{2}b^5) - 5/(2\sqrt{2}b^3)$. Substituting these equations into (4.3), we finally obtain $a(t) = (5/2)^{2/5}\, t^{3/5}$ ($t > 5/2$). It can be shown, that in the neighborhood of the point $t = 5/2$, the function a(t) is twice continuously differentiable, while its third derivative changes discontinuously at this instant [$a'''(5/2 + 0) - a'''(5/2 - 0) = 166/3125$].

Thus, after determining the function a(t), the plane problem of penetration of an ideal, slightly compressible fluid by a rigid contour is completely equivalent to the problem of flow past a slightly curved thin wing in supersonic gas flow. The latter problem has been studied in detail in [6], and the results obtained and the method of analysis are transferred to our problem without difficulty. In particular, simple formulas for calculating the pressure distribution along the wetted part of the contour were obtained. So in problem (1.11)-(1.16), the pressure distribution for $0 < t < 11/2$ is expressed by normal Lagrangian elliptic integrals of the first type.

The velocity field of the fluid parcels, computed in the acoustic approximation have integrable singularities in the neighborhood of the points of contact. These neighborhoods must be treated by the method of matched asymptotic expansions, and an "inner" expansion of the unknown functions constructed. This is done by considering the solution obtained in the acoustic approximation as the leading term of the outer asymptotic expansion of the solution for the original problem. As a preliminary, it should be stated that in a coordinate system moving with the point of contact, the fluid flow in a small neighborhood of this point will be essentially nonlinear and quasi-steady. The latter circumstance makes it possible to use the results of the theory of subsonic gas jets. The construction of an "inner" quasi-steady solution in this manner loses its validity at the instant the shock wave arrives at the free surface. This stage of the process is examined in [7]. It was shown that the motion of the fluid is described by a complex initial boundary value problem for the equations of transonic gas motion. However, for $M \to 0$, the duration of this stage tends toward zero, which is what makes it possible to consider the acoustic approximation without treating the transonic stage.

LITERATURE CITED

1. M. B. Lesser and J. E. Field, "The impact of compressible fluids," in: Ann. Rev. Fluid Mech., 15, 97 (1983).
2. A. A. Korobkin and V. V. Pukhnachov, "Initial stage of water impact," in: Ann. Rev. Fluid Mech., 20, 159 (1988).
3. V. V. Pukhnachev, "Linear approximation to the problem of the entry of a blunted body into water," Collection of Scientific Works of the Academy of Sciences of the USSR, Siberian Branch, Institute of Hydrodynamics, No. 38 (1979).
4. V. D. Kubenko, Penetration of a Compressible Fluid by Elastic Shells [in Russian], Naukova Dumka, Kiev (1981).
5. A. A. Korobkin, "Penetration of a slightly compressible fluid by a blunted body," Prikl. Mekh. Tekh. Fiz., No. 5 (1984).
6. E. A. Krasil'shchikova, A Thin Wing in Compressible Flow [in Russian], Nauka, Moscow (1986).
7. A. A. Korobkin and V. V. Pukhnachov, "Initial asymptotics in contact hydrodynamics problems," in: Proceedings of the 4th International Conference on Numerical Ship Hydrodynamics, S. 1, s. a., Washington (1985).